

# Inflato-Natural Leptogenesis: Leptogenesis in Chromo-Natural Inflation and Gauge-Flation

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In [1] a scenario of leptogenesis, the gravi-leptogenesis, was introduced. In this scenario the lepton asymmetry is created by the gravitational chiral anomaly within the standard model of particle physics, during inflation, for models of inflation driven by pseudoscalar field(s). In the gravi-leptogenesis scenario of [1], however, the pseudoscalar driven model of inflation was not specified. In this work we show that the inflation model can be successfully chosen to be the chromo-natural inflation [2] or the gauge-flation [3].

Significant improvement in the cosmological observations and their precision in the last couple of decades has enabled us to partially uncover history of the early Universe. The emerging picture is that the early Universe has undergone a period of accelerated expansion, inflationary period, followed by a reheating era leading to a radiation-dominated Universe and then matter-dominated period. Although we do not have the precise value of the inflationary scale  $H$  (Hubble during inflation) and the reheat temperature  $T_{\text{reh}}$ , current observations provide an upper bound of  $H \lesssim 10^{13}$  GeV and the strict lower bound of about  $T_{\text{reh}} \gtrsim 1$  MeV, due to the Big Bang Nucleosynthesis (BBN). Observation of primordial gravity waves by the Planck satellite will fix the value of  $H$ , while its non-observation will improve its upper bound.

A class of interesting questions one might ask is how details of inflationary model has affected the Universe we see today. Another class of interesting questions is how sensitive inflationary models are to the UV, in particular the Planck scale, physics. In this work we consider a particular model which has a feature relevant to the first class of questions while this question is asked within two models of inflation which have a better controlled UV sensitivity behavior than most of the usual inflationary models.

As far in the sky as we have observed, we seem not to have cosmological and astrophysical structures which are made out of antimatter; the observed Universe consists of matter rather than antimatter. The matter-antimatter asymmetry given by the observations is usually quoted as

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 5.5 \times 10^{-10}, \quad (1)$$

where  $n_B$ ,  $n_{\bar{B}}$  and  $n_\gamma$  are respectively the baryonic matter, antimatter and photon number densities in the observed Universe. Within the inflationary setups the standard lore is that even if the matter-antimatter asymmetry is given by the initial conditions of the Universe, this asymmetry is washed out by the rapid-accelerated expansion of the Universe. Therefore, one should seek a *dynamical* reasoning to explain the asymmetry.

About forty five years ago Sakharov [4] formulated the

three conditions needed for creating matter-antimatter asymmetry from symmetric initial conditions. Sakharov conditions demand existence of C and CP violation, baryon number violating interactions, and that these interactions should take place out of equilibrium. Within the particle physics setups, it is generically easier to first create a lepton asymmetry (leptogenesis) and then relay on thermally activated electroweak instantons (sphalerons) to create baryon asymmetry from the lepton asymmetry [5, 6]. The sphalerons would be activated if the temperature is not below  $\sim 1 - 10$  TeV. Therefore, standard leptogenesis models demand a reheat temperature of around  $T_{\text{reh}} \gtrsim 10$  TeV, which in itself and once we have a concrete reheating model, may impose a lower bound on  $H$ .

Despite the fact that out-of-equilibrium condition is granted, inflationary period is not usually fit for leptogenesis model building. This is because the mechanisms used to provide the other two of Sakharov conditions are not generically efficient enough to compensate for the wash-out effect (exponential dilution) caused by the rapid (usually almost exponential) expansion of the Universe during inflation. This obstacle, however, can be overcome if the mechanism for C and CP, and lepton number violation is based on the fields which are active during inflation, i.e. metric and the inflaton(s). This is the idea put forward in [1], the gravi-leptogenesis.

The gravi-leptogenesis is based on a particle physics model whose fermionic (chiral) matter content is assumed to be like that of the Standard Model (SM), with unequal number of left- and right- handed fermions. This model will have gravitational chiral anomaly on the  $B - L$  current and hence there is room for  $B - L$  violation if

$$R\tilde{R} = \frac{1}{2}\epsilon^{\alpha\beta\mu\nu}R_{\mu\nu\rho\sigma}R_{\alpha\beta}{}^{\rho\sigma} \quad (2)$$

is nonzero on the background, or has a nonvanishing vacuum expectation value. This latter can of course take place only if we have CP violation. (Within the standard model C violation is already built in.) Therefore, all the three Sakharov conditions can be readily met within this setup. So, what is computationally left is to provide the setting in which  $\langle R\tilde{R} \rangle$  is nonvanishing.

As discussed in [1]  $\langle R\tilde{R} \rangle$  will be nonzero in the models of inflation driven by a pseudoscalar inflaton and if the inflaton  $\chi$  has a  $P(\chi)R\tilde{R}$  coupling, where  $P(\chi)$  can be any odd function of  $\chi$ . In the discussions of [1] and in more detail [7, 8], it was argued that a  $P(\chi) = N\frac{\chi}{M_{\text{Pl}}}$  with  $N \sim 10^3$  naturally appears through supergravity or string theory compactifications involving axions. Nonetheless, a specific model for pseudoscalar driven inflation was not studied. In a sense the gravi-leptogenesis of [1] is a module which may be attached to an appropriate and successful inflation model.[19]

The first such appropriate choice may seem to be the natural inflation [10] in which the non-perturbatively induced axion potential drives inflation. This model has the well-discussed problem that in order to have successful slow-roll inflation the scale of the axion field should be super-Planckian. (The value of the potential is of course sub-Planckian.) In this sense, this model is not natural to usual non-Abelian gauge field theories in which the gauge fields are coupled to axion field  $\chi$  through the standard  $\frac{\chi}{f}F\tilde{F}$  term, where  $f$  is expected to be sub-Planckian (not bigger than GUT scales). Recently, the natural inflation model was revived under the title of chromo-natural inflation [2, 11], noting the role of non-Abelian gauge fields. Within the chromo-natural inflation model, although the energy budget driving inflation is still provided by the axion potential, the steepness of the axion potential, which is given by  $M_{\text{Pl}}/f$  in the absence of the gauge fields, is smoothed out and flattened through turning on a specific gauge field in the background.

In this work, we combine the gravi-leptogenesis and the chromo-natural inflation models to obtain the “inflato-natural leptogenesis” and use observed value of baryon asymmetry (1) to constrain further the parameter space of the chromo-natural model. As discussed in [11, 12] chromo-natural model and gauge-flation (inflation from non-Abelian gauge fields) model [3, 13] are closely related. We hence also discuss gravi-leptogenesis within the gauge-flation scenario.

### ***Inflato-Natural Leptogenesis, the Setup***

The inflato-natural leptogenesis consists of a pseudoscalar driven inflationary model and a gravitational anomaly part providing the source for parity and CP, as well as lepton number violation. In what follows we discuss each part separately.

#### ***Review of Chromo-Natural Inflation***

For the inflationary part we choose the chromo-natural inflation model of [2, 11]. The full Lagrangian of the model is hence

$$\mathcal{L} = -\frac{M_{\text{Pl}}^2}{2}R - \frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} + \frac{1}{2}(\partial_\mu \chi)^2 + \mu^4(1 + \cos \frac{\chi}{f}) + \frac{\lambda}{8f}\chi F\tilde{F} + P(\chi)R\tilde{R}. \quad (3)$$

Hereafter, we will work in units where  $M_{\text{Pl}}^{-2} = 8\pi G = 1$ .

The spacetime indices will be denoted by Greek letters while the gauge indices by small Latin indices  $a, b, c$ . Without loss of generality we choose the gauge group to be  $su(2)$  and hence  $a, b, c = 1, 2, 3$ . Inflation sector of the above Lagrangian has two dimensionless parameters gauge coupling  $g$  and axion-gauge field coupling  $\lambda$ , and two dimensionful parameters  $\mu$  and  $f$ . The axion-graviton coupling  $P(\chi)$  also includes one (or more) dimensionless parameters.

The inflationary FLRW trajectories with the metric convention,

$$ds^2 = -dt^2 + a(t)^2 dx_i dx_i = a(\tau)^2 (-d\tau^2 + dx_i dx_i),$$

are those with axion field  $\chi$  and certain component of the gauge field  $A_\mu^a$  in the temporal gauge [3, 13, 14]

$$A_\mu^a = \begin{cases} 0 & , \quad \mu = 0, \\ a\psi\delta_i^a & , \quad \mu = i, \end{cases} \quad (4)$$

turned on. With the above field configuration the rotation symmetry is retained, compensating the rotational non-invariance caused by turning on vector gauge fields in the background with the global part of the  $SU(2)$  gauge symmetry group. The  $\psi$  field is a scalar under spatial rotations. (For more detailed discussion see [13].) The slow-roll inflationary trajectories of the above model has been discussed in [11, 12]. For these trajectories  $\dot{\chi}/H\chi \sim \epsilon$ ,  $\dot{\psi}/H\psi \sim \epsilon^2$ , and during slow-roll inflation

$$\sin \frac{\chi}{f} \simeq \frac{3g\lambda}{\mu^4} H\psi^3, \quad \epsilon \simeq \psi^2 + \frac{3g^2\psi^4}{\mu^4(1 + \cos \frac{\chi}{f})}, \quad (5)$$

$$3H^2 \simeq \mu^4 \left(1 + \cos \frac{\chi}{f}\right), \quad \eta \simeq \psi^2,$$

where  $\epsilon$  and  $\eta$  are slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\ddot{H}}{2\dot{H}H}, \quad (6)$$

and  $\simeq$  stands for equality up to the first order in  $\epsilon$ ,  $\eta$ .

This model can lead to successful inflation for a wide range of its parameter space [15]. Two regions which have been studied more thoroughly are the small  $\chi$  region  $\chi_0/f \ll 1$  [11] and large  $\chi$  region  $\chi_0/f \sim \pi$  (for which the model coincides with the gauge-flation model) [12]. Typical value of parameters for these two regions are:

#### ***Small axion model [11]***

$$\begin{aligned} \chi_0 &= 5 \times 10^{-4}, & f &= 10^{-2}, & \lambda &= 200, \\ H &\simeq 8 \times 10^{-8}, & \mu^2 &= 10^{-7}, & g &= 2 \times 10^{-6}, \\ \psi &\simeq 1.7 \times 10^{-2}, & \epsilon &\simeq 3.5 \times 10^{-4}, & \eta &\simeq 3 \times 10^{-4}. \end{aligned} \quad (7)$$

#### ***Large axion model [12]***

$$\begin{aligned} \chi_0 - \pi f &= 5 \times 10^{-4}, & f &= 10^{-2}, & \lambda &= 2 \times 10^4, \\ H &\simeq 3.3 \times 10^{-5}, & \mu^2 &= 1.6 \times 10^{-3}, & g &= 10^{-3}, \\ \psi &\simeq 4 \times 10^{-2}, & \epsilon &\simeq 4 \times 10^{-3}, & \eta &\simeq 1.6 \times 10^{-3}. \end{aligned} \quad (8)$$

### Gravitational Anomaly, Lepton and CP violation

It is well-established in the literature that the gravitational chiral anomaly is

$$\partial_\mu J_\ell^\mu = \frac{\mathcal{A}}{16\pi^2} R\tilde{R} \quad (9)$$

where  $\mathcal{A} = n_L - n_R$  measures the difference between number of left- and right-handed fermion degrees of freedom. For standard model  $\mathcal{A} = 3$ , while for beyond standard models with right-handed neutrinos it could be less than three. It was argued in [1] that the pseudoscalar inflaton field  $\chi$  with  $P(\chi)R\tilde{R}$  coupling, induces gravitational birefringence and hence a nonzero  $\langle R\tilde{R} \rangle$ . This will in turn lead to a nonzero result for the integral of the left hand side of (9) which is nothing but the lepton number asymmetry created by the anomaly during inflation.

As we show below the  $\chi$  field of chromo-natural inflation can be taken as the pseudoscalar field in the analysis of [1]. However, we make a comment before showing some calculations. In the chromo-natural inflation model, and similarly in the gauge-flation model [13], the gauge field turned on in the background has a nonzero  $F\tilde{F} \propto (\dot{\psi} + H\psi)\psi^2$ , which is of course proportional to  $\sin \chi/f$ , cf. (5).  $F\tilde{F}$  is proportional to gauge field instanton number density (though in the Lorentzian signature) which is a source for P and CP violation. In this sense inflation, and also the nonzero  $\langle R\tilde{R} \rangle$ , are both driven by  $F\tilde{F}$ . During the slow-roll inflation the “instanton number density” varies slowly and its time variation pushes inflation toward the end.

To show explicitly that the analysis of [1] also holds for the case with both  $\chi$  and gauge field, following [1], we study equation of motion for the left and right circular gravity wave polarizations. The equation of motion for left-handed gravitons is

$$\square h_L = -\frac{8i}{a^3} \left( a^2 \dot{P} \dot{h}'_L \right)' = -\frac{8i}{a} \left[ (2H\dot{P} + \ddot{P}) \dot{h}'_L + \dot{P} \ddot{h}'_L \right], \quad (10)$$

where dot denotes derivative w.r.t. the comoving time  $t$  and prime is derivative w.r.t. the spatial dependence, which may be taken to be the  $x^3 = z$  direction. The equation for right mode is complex conjugate of the above and hence we have gravitational birefringence once the right-hand-side of the above equation is nonzero [16]. The term with third order derivatives of  $h_L$  may be omitted once we compare it, in the Fourier space, with similar time derivatives of  $h_L$  appearing in  $\square h_L$ , i.e.

$$\frac{a^{-1} \dot{P} \ddot{h}'_L}{\ddot{h}_L} \sim \frac{k\dot{\chi}}{a} \frac{\partial P}{\partial \chi}. \quad (11)$$

To estimate the above ratio we may choose a physically motivated function  $P(\chi)$ . One such choice which is generic to string theory compactifications is [1, 8]

$$P = \frac{\mathcal{N}}{16\pi^2} \frac{\chi}{\mathcal{F}}, \quad (12)$$

where  $\mathcal{N}$  is of order  $10^3 - 10^4$  and  $\mathcal{F}$  is the string scale. One expects a hierarchy

$$\mu \lesssim f, \Lambda \lesssim \mathcal{F} \lesssim M_{\text{pl}}, \quad (13)$$

$\Lambda$  being the gravitational cutoff. Later we will discuss two extremes: (i)  $\mathcal{F} = M_{\text{pl}}$ , and (ii)  $\mathcal{F} = \Lambda$ . For our model, using (5), we have

$$\frac{\dot{\chi}}{Hf} = -\epsilon \tan \frac{\chi}{f}, \quad (14)$$

and hence

$$\frac{k\dot{\chi}}{a} \frac{\partial P}{\partial \chi} \simeq -\frac{\mathcal{N}\epsilon}{16\pi^2} \frac{k}{aM_{\text{pl}}} \frac{H}{M_{\text{pl}}} \frac{f}{\mathcal{F}} \tan \frac{\chi}{f} \ll 1, \quad (15)$$

since each fraction in the above is less than one.

Noting that for the slowly rolling  $\chi$ ,  $\ddot{\chi}/H\dot{\chi} \ll 1$ , one may further approximate (10) to obtain

$$\square h_L = -2i \frac{\Theta}{a} \dot{h}'_L, \quad (16)$$

where

$$\Theta \simeq -\frac{\mathcal{N}\epsilon}{2\pi^2} \left( \frac{H}{M_{\text{pl}}} \right)^2 \frac{f}{\mathcal{F}} \tan \frac{\chi}{f}. \quad (17)$$

### Lepton asymmetry, $n/s$ ratio

Lepton number asymmetry is computed by inserting solutions of (16) into the right-hand-side of anomaly equation, computing the vacuum expectation value of  $R\tilde{R}$ , and integrating over the space. We note that in this computation *quantum* subhorizon modes with  $k > aH$  should be considered. Since we are dealing with a UV divergent integral we need to regularize it which is done by a cutoff  $\Lambda$ , where  $\Lambda \gg H$ . [20] The details of the computations relating the lepton number asymmetry to the left-right asymmetry in gravity wave sector has been given in [1]. Here we just quote the result for the density of lepton number asymmetry  $n$

$$n = \frac{1}{72\pi^4} \left( \frac{H}{M_{\text{pl}}} \right)^2 |\Theta| H^3 \left( \frac{\Lambda}{H} \right)^6. \quad (18)$$

To compare with the observed data we need to compute the entropy density of the Universe too. We do this with the standard assumption that the entropy of the Universe has not changed since the end of reheating. Next we need a reheating model. Here we simply assume a slightly improved instant reheating model with a single “refining” parameter  $\sigma$ :

$$\rho_{\text{reheat}} = \sigma \rho_0 = \frac{\pi^2}{30} g_* T^4, \quad (19)$$

where  $g_*$  is the number of relativistic degrees of freedom and  $\rho_0 = 3H^2 M_{\text{pl}}^2$  is the energy density during inflation.  $\sigma$  is a parameter which measures the “efficiency” of the

reheating process. For instant reheating,  $\sigma = 1$  (100% efficiency) which leads to a typical reheat temperature  $T \sim 10^{14}$  GeV. Within supergravity models it is usually required to have reheat temperatures below  $10^9$  GeV to avoid gravitino overproduction problem (see e.g. [17] for more detailed discussion); this happens if  $\sigma \lesssim 10^{-20}$ . The entropy density  $s$  is then

$$s = \frac{2\pi^2}{45} g_* T^3 = 2.3 g_*^{1/4} \sigma^{3/4} (H M_{\text{pl}})^{3/2}. \quad (20)$$

Finally, we can compute the desired  $n/s$

$$\begin{aligned} \frac{n}{s} &= 6 \times 10^{-5} g_*^{-1/4} \sigma^{-3/4} \left( \frac{H}{M_{\text{pl}}} \right)^{7/2} |\Theta| \left( \frac{\Lambda}{H} \right)^6 \quad (21) \\ &= 3 \times 10^{-6} g_*^{-1/4} \sigma^{-3/4} \mathcal{N} \epsilon \sqrt{\frac{M_{\text{pl}}}{H}} \left( \frac{\Lambda}{M_{\text{pl}}} \right)^6 \frac{f}{\mathcal{F}} |\tan \frac{\chi}{f}|, \end{aligned}$$

where in the second line (17) has been used. Recalling the analysis of [5] and the observations, the above should be compared with the observed value  $n/s = 2.4 \times 10^{-10}$ .

Inserting the typical values of the *small axion* chromo-natural inflation model (7) we obtain, for  $\mathcal{F} = M_{\text{pl}}$ ,

$$\frac{n}{s} = 1.86 \times 10^{-9} g_*^{-1/4} \sigma^{-3/4} \mathcal{N} \left( \frac{\Lambda}{M_{\text{pl}}} \right)^6. \quad (22)$$

For typical values of  $g_* \sim 10^2$  and  $\mathcal{N} \sim 10^3$ , the above leads to successful leptogenesis model if  $\Lambda \sim 0.3 \sigma^{1/8} M_{\text{pl}}$ . For  $\sigma \sim 10^{-20}$ , that is  $\Lambda \sim \mu \sim 10^{15}$  GeV (*cf.* (7)). Using  $\mathcal{F} = \Lambda$  we find  $\Lambda \sim 0.2 \sigma^{3/20} M_{\text{pl}}$ , leading to  $\Lambda \sim 10^{15}$  GeV again. Recalling that  $\mu$  itself is the natural cutoff scale for the axion theory, we obtain the appealing result that the cutoff of the gravity theory coincides  $\mu$  and hence we have a single natural scale in our model.

One may explore if the *large axion* region (8), i.e., with  $\chi_0 \simeq \pi f$  where the model coincides with the gauge-flation, can also lead to a successful inflato-natural leptogenesis. It is readily seen that for the “gauge-flation leptogenesis” case one can still use (21) but now replacing  $\tan \frac{\chi}{f}$  by its typical value 0.05. Using the typical values given in (8), and  $g_* \sim 10^2$ ,  $\mathcal{N} \sim 10^3$ , one again obtains the previous results for both values of  $\mathcal{F}$ , i.e.,  $\Lambda \sim 10^{15}$  GeV. In this case, however,  $\mu \sim 10^{17}$  GeV. Although  $\Lambda$  is two orders of magnitude smaller than  $\mu$  which makes the model less appealing than the small axion case, it is still theoretically viable since  $\Lambda \gtrsim H$ .

As we showed the ratio  $n/s$  in our model crucially depends on the reheating temperature. In our analysis we phenomenologically parametrized the efficiency of the reheating model by the  $\sigma$  parameter. It is desirable to study in more detail reheating within our gauge-flation and/or chromo-natural model. In fact, as discussed in [11, 12] the two models become identical in the end of inflation. Reheating in these models is also natural in the sense that the energy of the system is already in the

coherent oscillations of the gauge fields which could be taken to be gauge fields of any beyond standard model and hence energy can directly be transferred to other standard model particles through gauge interactions.

Finally we would like to mention the interesting possibility of realizing asymmetric dark matter scenarios [18] within the inflato-natural leptogenesis. The main theme of the asymmetric dark matter models is that the dark matter particle  $X$ , which ought to be a stable neutral particle, is produced through the same mechanism as leptogenesis and the asymmetry forbids  $X \bar{X}$  annihilation. To realize this model, however, one needs to consider beyond standard model. Exploring this idea is postponed to future works.

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  - [19] See [9] for another leptogenesis-during-inflation model where coupling of axion to Abelian Chern-Simons gauge fields (instead of  $R\bar{R}$  coupling) is driving the leptogenesis.

[20] In the notation of [1] this cutoff was called  $\mu$ . Here, since we have used  $\mu$  as the coefficient in front of the axion

cosine potential, we use  $\Lambda$  for the cutoff.